Math Circle 10/4/08

POLYNOMIALS

1) Find all the polynomials P(x) that satisfy the equality

$$P(x^2) = (P(x))^2 .$$

2) Polynomial P(x) has remainder 2 when divided by x-1, and remainder 1 when divided by x-2. What is the remainder of this polynomial when it is divided by (x-1)(x-2)?

3) What is the condition that numbers *a,b* need to satisfy so that the polynomial $p(x) = x^5 + ax^3 + b$ has at least one double root that is not zero.

4) Prove that $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ can not be written as a product of two polynomials with integer coefficients.

5) Find all pairs of integers *(p,q)* such that all the zeros of polynomials $x^2 + px + q$ and $x^2 + qx + p$ are integers.

6) For integers *a,b,c* we know that *a*>0 and that polynomial $ax^2 + bx + c$ has two distinct zeros in the interval (0,1). Prove that $a \geq 5$, Find one pair of *b,c* for *a*=5.

7) Find all the pairs of positive integers *x,n* that satisfy the equation $x^3 + 2x + 1 = 2^n$

8) How many real zeros does the polynomial $x^4 - 2x^3 - 4x^2 + 5x + 5$ have? (No calculators please)

9) Calculate the sum of the squares of the solutions of the cubic equation $x^3 + 2x - 3 = 0$

10) Prove that quadratic equations $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$, $a,b,c \in R$, $a,b \neq 0$ have a common solution if and only if $a^3 + b^3 + c^3 = 3abc$.

11) Find the values of λ for which the polynomials $f(x) = x^3 - \lambda x + 2$ and $g(x) = x^2 + \lambda x + 2$ have at least one common root.

12) Find the values of λ, μ for which the polynomials $f(x) = x^3 + 5x^2 + \lambda x + 2$ and $g(x) = x^3 + \mu x - 8$ have at least two common roots.